



CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

9231/23

May/June 2021

2 hours

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

1 (a) Find a and b such that

$$z^8 - iz^5 - z^3 + i = (z^5 - a)(z^3 - b). \quad [1]$$

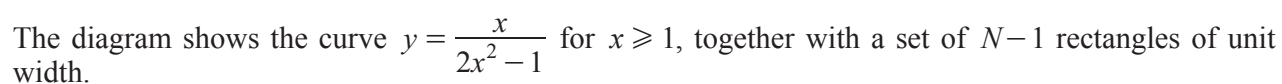
(b) Hence find the roots of

$$z^8 - iz^5 - z^3 + i = 0,$$

giving your answers in the form $re^{i\theta}$, where $r > 0$ and $0 \leq \theta < 2\pi$. [6]

- 2** Find the Maclaurin's series for $\ln \cosh x$ up to and including the term in x^4 . [7]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.


$$\sum_{r=1}^N \frac{r}{2r^2-1} < \frac{1}{4} \ln(2N^2-1) + 1. \quad [7]$$
[illegible]

- (b) Use a similar method to find, in terms of N , a lower bound for $\sum_{r=1}^N \frac{r}{2r^2 - 1}$. [3]

- 4 By considering the binomial expansions of $\left(z + \frac{1}{z}\right)^5$ and $\left(z - \frac{1}{z}\right)^5$, where $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that

$$\tan^5 \theta = \frac{\sin 5\theta - a \sin 3\theta + b \sin \theta}{\cos 5\theta + a \cos 3\theta + b \cos \theta},$$

where a and b are integers to be determined.

[7]

[illegible]

- 5 The variables x and y are related by the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 4e^{-x}.$$

- (a) Find the value of the constant k such that $y = kxe^{-x}$ is a particular integral of the differential equation. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the solution of the differential equation for which $y = \frac{dy}{dx} = \frac{1}{2}$ when $x = 0$. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 6 (a) Starting from the definitions of $\sinh$ and $\cosh$ in terms of exponentials, prove that

$$2 \sinh^2 x = \cosh 2x - 1. \quad [3]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the solution to the differential equation

$$\frac{dy}{dx} + y \coth x = 4 \sinh x$$

for which $y = 1$ when $x = \ln 3$. [7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

7 The integral I_n , where n is an integer, is defined by $I_n = \int_0^{\frac{3}{2}} (4+x^2)^{-\frac{1}{2}n} dx$.

(a) Find the exact value of I_1 , expressing your answer in logarithmic form. [3]

[illegible]

(b) By considering $\frac{d}{dx} \left(x(4+x^2)^{-\frac{1}{2}n} \right)$, or otherwise, show that

$$4nI_{n+2} = \frac{3}{2}\left(\frac{2}{5}\right)^n + (n-1)I_n. \quad [5]$$

[illegible]

- (c) Find the value of I_5 . [3]

- 8 (a)** Find the value of a for which the system of equations

$$\begin{aligned} 13x + 18y - 28z &= 0, \\ -4x - ay + 8z &= 0, \\ 2x + 6y - 5z &= 0, \end{aligned}$$

does not have a unique solution.

[2]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 13 & 18 & -28 \\ -4 & -1 & 8 \\ 2 & 6 & -5 \end{pmatrix}.$$

- (b) Find the eigenvalue of $\mathbf{A}$ corresponding to the eigenvector $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$. [1]

- (c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A} = \mathbf{PDP}^{-1}$. [8]

- (d) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$ in terms of $\mathbf{A}$. [2]

[illegible]

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.